INTERACTIONS FAIBLES : REVUE

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Nous allons passer très rapidement en revue les propriétés connues des interactions faibles qui nous serviront de guide pour la construction des modèles. Ces notes n'ont pas pour but d'enseigner la théorie des interactions faibles et ne remplacent pas un cours à ce sujet.

1. INTERACTIONS LEPTONIQUES

1.1 Spectre des leptons

Nous connaissons actuellement quatre états leptoniques ainsi que leurs antiparticules. Ils font intervenir deux nombres quantiques additifs conservés séparément : le nombre "muonique" (M) et le nombre "électronique" (E). Les différentes propriétés des quatre leptons sont indiquées dans le tableau suivant.

	μ^-	ν_μ	e^-	ν_e
Masse (MeV)	105,6595	< 0,65	0,5110041	$< 6 \times 10^{-5}$
Q	-1	0	-1	0
M	1	1	0	0
E	0	0	1	1

Remarques :

a) Y a-t-il d'autres états leptoniques? (Leptons lourds?)

b) Les expériences neutrino nous ont déjà montré l'existence de deux neutrinos distincts ν_μ et ν_e. Jusqu'à présent, nous ne sommes cependant pas certains que tous les neutrinos connus appartiennent à l'une de ces deux catégories. (Pour une revue des différentes possibilités, voir S.L. Glashow, Rencontres de Moriond, 1971.)

c) Nous supposerons que les masses de deux neutrinos sont égales à zéro, même si les limites, surtout pour ν_μ, sont assez grandes.

d) L'hélicité de tous les neutrinos est "gauche" ("left").

1.2 Interactions

La seule interaction leptonique étudiée expérimentalement est la désintégration du μ :

$$\mu^- \longrightarrow e^- + \nu_\mu + \bar{\nu}_e \, . \tag{1}$$

Elle peut être décrite complètement par un Lagrangien phénoménologique de la forme :

$$\mathcal{L} = \bar{\mu}(x)\left(i\partial\!\!\!/ - m_\mu\right)\mu(x) + \bar{\nu}_\mu(x)\, i\partial\!\!\!/\, \nu_\mu(x) + \bar{e}(x)\left(i\partial\!\!\!/ - m_e\right)e(x)$$
$$+ \bar{\nu}_e(x)\, i\partial\!\!\!/\, \nu_e(x) + \frac{G}{\sqrt{2}}\left[\bar{\mu}(x)\gamma_\lambda(1+\gamma_5)\nu_\mu(x)\right]\left[\bar{\nu}_e(x)\gamma^\lambda(1+\gamma_5)e(x)\right] + h.c. \tag{2}$$

Nous avons utilisé les mêmes lettres pour représenter les particules et leurs champs. Les quatre premiers termes forment le Lagrangien libre, le dernier celui d'interaction. $G/\sqrt{2}$ est la constante de couplage de Fermi : $G/\sqrt{2} \sim 10^{-5}/m^2_{proton}$. h.c. représente l'hermitique conjugué du terme d'interaction.

Si nous définissons pour les deux neutrinos

$$\nu_{\substack{R\\L}}(x) = \frac{1}{2}\left(1 \mp \gamma_5\right)\nu(x) \, , \tag{3}$$

nous constatons que seuls les neutrinos "left" $\nu_{\mu L}$ et ν_{eL} sont couplés. Les composantes "right" $\nu_{\mu R}$ et ν_{eR} peuvent être omises même du Lagrangien libre.

On peut généraliser la formule (2). Définissons un "courant leptonique" ℓ_λ par analogie avec le courant électromagnétique :

$$\ell_\lambda(x) = \bar{\mu}(x)\gamma_\lambda \nu_{\mu_L}(x) + \bar{e}(x)\gamma_\lambda \nu_{e_L}(x) \, . \tag{4}$$

Nous pouvons donc écrire un Lagrangien d'interaction effectif :

$$\mathcal{L}_{eff.} = \frac{G}{\sqrt{2}}\, \ell^\lambda(x)\, \ell^\dagger_\lambda(x) \, . \tag{5}$$

L'interaction (5) prédit, en plus de (1), d'autres processus leptoniques comme, par exemple, $\nu_e + e^- \to \nu_e + e^-$. L'observation de ces processus et la détermination expérimentale de leurs constantes de couplage constituent donc un test important de la théorie des interactions faibles.

Remarques :

a) Y a-t-il d'autres "courants"? On pense à l'événement unique observé dans Gargamelle et qui semble représenter le processus $\nu_\mu + e^- \to \nu_\mu + e^-$. Ce processus est absent du Lagrangien (5). On en reparlera plus loin.

b) Au niveau du Lagrangien (5), la forme des courants n'est pas bien définie parce qu'on a la possibilité d'effectuer des transformations de Fierz, qui permutent les différents champs des fermions. La forme des courants devient inambiguë par l'introduction d'un boson vectoriel intermédiaire (voir plus loin).

c) Si nous négligeons toutes les masses, y compris celles du μ^- et du e^-, $\ell_\lambda(x)$ est conservé : $\partial_\lambda \ell^\lambda(x) = 0$. Par conséquent, on peut définir une charge conservée Q_ℓ ainsi que son hermitique conjuguée Q_ℓ^+. Leur commutateur $[Q,Q^+]$ définit une charge électriquement neutre conservée Q^0. Q, Q^+ et Q^0 sont les générateurs d'un groupe SU(2).

2. INTERACTIONS SEMI-LEPTONIQUES

2.1 $\Delta S = 0$

Les désintégrations β, les désintégrations des mésons π, la diffusion des neutrinos sur les nucléons, etc., sont des exemples de ce type de processus.

Tous les résultats expérimentaux peuvent de nouveau être décrits par un Lagrangien phénoménologique de la forme courant × courant. Pour cela, nous introduisons un courant hadronique $h_\lambda^{(0)}(x)$ qui contient une partie vectorielle et une partie axiale sous la forme V-A. L'interaction phénoménologique s'écrit alors :

$$\mathcal{L}_{eff} = \frac{G}{\sqrt{2}} \ell^\lambda(x) h_\lambda^{(0)+}(x) + h.c. \tag{6}$$

Considérons l'amplitude de la désintégration β du neutron. La partie hadronique est donnée par :

$$\sqrt{\frac{p_0 \, p_0'}{m^2_{nucléon}}} \langle p | h_\lambda^{(0)}(0) | n \rangle =$$

$$= i \, \bar{u}_{proton}(p) \Big\{ g_V(q^2) \gamma_\lambda + f_T(q^2) \sigma_{\lambda\rho} q^\rho + g_A(q^2) \gamma_\lambda \gamma_5$$

$$+ i f_P(q^2) q_\lambda \gamma_5 \Big\} u_{neutron}(p'), \qquad (7)$$

où g_V, f_T, g_A et f_P sont des facteurs de forme qui dépendent du carré du moment de transfert $q = p - p'$. A $q = 0$, nous avons besoin uniquement des facteurs de forme vectoriel et axial $g_V(0)$ et $g_A(0)$. Expérimentalement, $g_V(0) \sim 1$, et $g_A(0) \sim 1{,}2$. Cela signifie que la constante de couplage du courant vectoriel de la désintégration β est égale à celle de la désintégration du μ. Ce résultat remarquable est à l'origine de l'hypothèse du courant vectoriel conservé. En effet, le facteur de forme $g_V(0)$ représente les effets de renormalisation dus aux interactions fortes. *A priori*, on s'attendait à une valeur quelconque; mais le résultat $g_V(0) = 1$ montre l'absence de ces effets de renormalisation.

Il existe un autre cas en phyqique où ces effets sont absents, à savoir la charge électrique. Le positron et le proton ont la même charge électrique malgré le fait que le proton a des interactions fortes et que le positron n'en a pas. Il est bien connu que cette universalité de la charge électrique est due à l'existence du courant électromagnétique conservé. Il est donc naturel de supposer que le courant vectoriel faible est aussi conservé : c'est l'hypothèse du CVC (courant vectoriel conservé). Cette hypothèse peut même aller plus loin : on peut en effet essayer de lier ces deux courants vectoriels conservés, l'électromagnétique et le faible, en postulant que la partie isovectorielle du courant électromagnétique et la partie vectorielle du courant faible à $\Delta S = 0$ forment un triplet du groupe du spin isotopique. Il en résulte qu'on peut définir un triplet des charges conservées $Q^i (i = 1, 2, 3)$ qui sont les générateurs des transformations du groupe de l'isospin. Leurs lois de commutation sont donc données par :

$$[Q^i, Q^j] = i\epsilon_{ijk} Q^k \tag{8}$$

où

$$Q^i = \int d^3x \, V_0^i(\vec{x}, t) \tag{9}$$

$$\partial^\mu V_\mu^i(x) = 0 . \tag{10}$$

Jusqu'à maintenant, nous nous sommes préoccupés de la partie vectorielle $V_\mu(x)$ du courant faible. Regardons maintenant la partie axiale A_μ. On serait tenté de postuler une loi de conservation analogue à (10), mais une telle hypothèse se trouverait en contradiction flagrante avec l'expérience. D'abord $g_A(0) \neq 1$, comme nous l'avons dit plus haut; ensuite, les résultats seraient catastrophiques en ce qui concerne les désintégrations du méson π. En effet, l'amplitude de la désintégration $\pi^+ \to \mu^+ + \nu_\mu$ est proportionnelle à

$$\langle 0 | A_\lambda(0) | \pi \rangle \sim f_\pi q_\lambda , \tag{11}$$

où q est le moment du π et f_π une constante.

De (11) nous obtenons :

$$\langle 0 | \partial^\lambda A_\lambda(0) | \pi \rangle \sim f_\pi m_\pi^2 \neq 0 . \tag{12}$$

Par conséquent, l'hypothèse du courant axial conservé $\partial^\lambda A_\lambda = 0$ entraîne $f_\pi = 0$ (pion stable!) ou $m_\pi = 0$ (pion de masse nulle). C'est la seconde possibilité ($m_\pi = 0$) qui est considérée dans le cadre des théories d'invariance chirale, avec l'hypothèse selon laquelle un monde contenant des pions de masse nulle constitue une bonne approximation du monde où nous vivons. Dans ce dernier, $f_\pi m_\pi^2 \neq 0$, et par conséquent le courant axial n'est pas conservé. On en déduit la présence des effets de renormalisation, d'où $g_A(0) \neq 1$. On se trouve maintenant en présence d'un problème très grave. Comme ces effets de renormalisation dus aux interactions fortes ne sont pas calculables, rien ne distingue une théorie du type V-A d'une autre du type V-αA, où α est un nombre quelconque. Par conséquent, la notion même de l'universalité du couplage des interactions faibles est mise en cause.

Il nous faut un moyen indépendant pour déterminer sans ambiguïté l'échelle du courant axial par rapport au courant vectoriel. Ce moyen est fourni par l'algèbre des courants : Supposons d'abord que le courant axial fait aussi partie d'un triplet du spin isotopique $A^i_\lambda(x)$ $(i = 1, 2, 3)$. D'une façon formelle, nous pouvons définir un triplet des charges $Q^i_5(t)$ par l'analogue des équations (9) :

$$Q_5^i(t) = \int d^3x\, A_0^i(\vec{x},t), \tag{13}$$

la différence étant que $A^i_\mu(x)$ n'est pas conservé et que, par conséquent, les charges correspondantes dépendent du temps. L'équation (13), ainsi que toutes les autres qui contiennent les charges axiales, sont supposées être valables uniquement en tant qu'éléments de matrices entre états physiques. Puisque $Q^i_5(t)$ forment un triplet, leurs lois de commutation avec les générateurs Q^i sont connues :

$$\left[Q^i, Q_5^j(t)\right] = i\,\epsilon_{ijk}\, Q_5^k(t). \tag{14}$$

Or cette relation est linéaire en Q_5, et par conséquent elle ne permet pas de déterminer l'échelle relative entre Q_5 et Q. La relation non linéaire cherchée est fournie par le commutateur de deux charges axiales. L'hypothèse fondamentale de l'algèbre des courants est que le commutateur de deux charges axiales, prises à temps égaux, est égal à celui de deux charges vectorielles :

$$\left[Q_5^i(t), Q_5^j(t)\right] = \left[Q^i, Q^j\right] = i\,\epsilon_{ijk}\, Q^k. \tag{15}$$

Les équations (8), (14) et (15) montrent que les six opérateurs Q^i et $Q^i_5(t)$ sont, à temps égaux, les générateurs de l'algèbre du groupe $SU(2) \times$ $\times\, SU(2)$. Cette structure algébrique s'est révélée riche en prédictions et a conduit à une compréhension plus profonde des lois de symétrie des particules élémentaires. Sa connaissance est indispensable pour une étude détaillée des nouvelles théories de jauge des interactions faibles, mais elle sort du cadre du présent exposé.

2.2 $\Delta S = 1$

Les désintégrations leptoniques des hypérons, celles des mésons K, les expériences neutrino avec changement d'étrangeté, etc., sont des exemples de ce type de processus.

Encore une fois, tous les résultats expérimentaux peuvent être décrits par l'introduction d'un nouveau courant hadronique $h_\lambda^{(1)}(x)$, qui change l'étrangeté d'une unité.

Quelle est la constante de couplage du nouveau courant avec le courant leptonique? Expérimentalement, il semble que $h_\lambda^{(1)}(x)$ est couplé moins fortement que $h_\lambda^{(0)}(x)$. Cela signifie-t-il que la notion de l'universalité des interactions faibles, qui nous a tellement aidés à découvrir des lois de symétrie au chapitre précédent, doit être abandonnée? La réponse à cette question nous conduira à la théorie de Cabibbo.

Afin de faciliter l'exposé, nous utiliserons le langage du modèle des quarks, étant entendu que les résultats ne dépendent point de l'existence de particules élémentaires du genre des quarks. Les champs des quarks q(x) sont utilisés uniquement pour expliciter les lois de transformation des courants.

Soit donc q(x) le champ d'un triplet avec les nombres quantiques habituels des quarks :

$$q(x) = \begin{pmatrix} p(x) \\ n(x) \\ \lambda(x) \end{pmatrix} . \tag{16}$$

Les courants faibles hadroniques que nous avons introduits sont représentés par :

$$h_\mu^{(0)}(x) = \bar{p}(x)\gamma_\mu(1+\gamma_5)n(x) \tag{17}$$

$$h_\mu^{(1)}(x) = \bar{p}(x)\gamma_\mu(1+\gamma_5)\lambda(x) . \tag{18}$$

Le courant hadronique total est

$$h_\mu(x) = \alpha\, h_\mu^{(0)}(x) + \beta\, h_\mu^{(1)}(x) , \tag{19}$$

où α et β sont des nombres qui déterminent la force relative de couplage entre les deux courants.

L'équation (19) s'écrit alors :

$$h_\mu(x) = \bar{p}(x)\gamma_\mu(1+\gamma_5)\left[\alpha\, n(x) + \beta\, \lambda(x)\right]$$

$$= \sqrt{\beta^2+\alpha^2}\;\bar{p}(x)\gamma_\mu(1+\gamma_5)\left[\cos\theta\, n(x) + \sin\theta\, \lambda(x)\right] \qquad (20)$$

avec tg $\theta = \beta/\alpha$.

Nous pouvons maintenant écrire, dans le langage du modèle des quarks, le Lagrangien total de toutes les interactions sous la forme :

$$\mathcal{L} = i\,\bar{q}(x)\,\not{\partial}\,q(x) - \bar{q}(x)\,M\,q(x) + \mathcal{L}_{fortes}$$

$$+ \bar{q}(x)\,Q\,\gamma_\mu\, q(x)\,A^\mu(x)$$

$$+ \frac{G}{\sqrt{2}}\,\ell^\mu(x)\,h_\mu^\dagger(x) + h.c. + \dots \qquad (21)$$

où M est la matrice de masse et Q celle de charge des quarks, données par :

$$M = \begin{pmatrix} m_p & & 0 \\ & m_n & \\ 0 & & m_\lambda \end{pmatrix} \qquad Q = e\begin{pmatrix} \frac{2}{3} & & 0 \\ & -\frac{1}{3} & \\ 0 & & -\frac{1}{3} \end{pmatrix}. \qquad (22)$$

$\mathcal{L}_{fortes}$ est le Lagrangien des interactions fortes, $A_\mu(x)$ est le champ du proton et $\ell_\mu(x)$ le courant leptonique donné par (4). Les points à la fin représentent les termes que nous n'avons pas écrits explicitement, tels que le Lagrangien libre du photon et des leptons, les interactions électromagnétiques des leptons, les interactions faibles non leptoniques et celles purement leptoniques, etc.

Supposons maintenant que l'on néglige les interactions moyennement fortes qui violent SU(3). Alors toute la première ligne de (21) conserve SU(3) et, par conséquent, toutes les masses des quarks sont égales : M est proportionnelle à la matrice unité. On peut alors effectuer une transformation de SU(3) sur les champs des quarks $q \to q'$:

$$q'(x) = \begin{pmatrix} p(x) \\ n(x)\cos\theta + \lambda(x)\sin\theta \\ -n(x)\sin\theta + \lambda(x)\cos\theta \end{pmatrix} \equiv \begin{pmatrix} p(x) \\ n_c(x) \\ \lambda_c(x) \end{pmatrix} . \tag{23}$$

Tous les termes de (21) sont alors invariants sous la transformation (23), à l'exception du dernier terme, qui représente les interactions faibles semi-leptoniques et qui devient :

$$\frac{G}{\sqrt{2}}\, \ell^{\mu}(x)\, \bar{p}(x)\gamma_{\mu}(1+\gamma_5)\, n_c(x) + h.c. \tag{24}$$

Si SU(3) était conservé par les interactions fortes, $q'(x)$ serait une base physiquement aussi acceptable que $q(x)$. Dans ce cas, l'expression (24) nous montre que, en termes des champs $q'(x)$, les interactions faibles conservent l'"étrangeté"; seulement, il s'agit de l'étrangeté définie par $\lambda_c(x)$ et non de celle définie par $\lambda(x)$. Il faut cependant répéter que, si SU(3) est conservé à la première ligne de (21), les interactions fortes n'ont aucune raison de préférer l'une plutôt que l'autre. On serait donc amenés à définir d'une façon naturelle l'étrangeté comme le nombre quantique du quark $\lambda_c(x)$ et ce nombre quantique serait conservé par toutes les interactions. La base naturelle serait la base $q'(x)$ et il n'y aurait pas d'angle dans le Lagrangien des interactions faibles.

Tout cela est vrai pour le monde hypothétique où SU(3) est conservé par les interactions fortes; mais dans le monde tel que nous le connaissons, ceci n'est pas vrai : SU(3) est violé par les interactions dites moyennement fortes. Il en résulte que M n'est plus proportionnelle à la matrice unité, mais a la forme générale (22) avec $m_n \neq m_\lambda$. Dans ce cas, la transformation (23) n'est plus une symétrie. Les interactions fortes ont choisi une base privilégiée, la base $q(x)$, et c'est par rapport à elle qu'on doit écrire toutes les autres interactions.

Récapitulons : les interactions moyennement fortes ont une base privilégiée et définissent ainsi une étrangeté, représentée par $\lambda(x)$, qu'elles conservent. D'autre part, les interactions faibles préfèrent une autre base, $q'(x)$, et conservent ainsi une autre étrangeté, définie par $\lambda_c(x)$. Les deux bases sont tournées l'une par rapport à l'autre d'un angle qui est l'angle de Cabibbo. C'est l'angle du désaccord entre les directions choisies par les interactions fortes et faibles. Comme les interactions fortes sont de loin les plus fortes, il est naturel de retenir leur définition comme définition de l'étrangeté. Elle est violée par les interactions faibles selon (20).

L'universalité des interactions faibles est restaurée maintenant en postulant que :

$$\sqrt{\beta^2+\alpha^2} = 1$$

et le courant hadronique s'écrit comme :

$$h_\mu(x) = \bar{p}(x)\gamma_\mu(1+\gamma_5)\left[n(x)\cos\theta + \lambda(x)\sin\theta\right]. \tag{25}$$

$h_\mu(x)$ définit aussi un groupe SU(2), et toutes les considérations sur l'algèbre des courants s'étendent directement ici.

Les principales propriétés du courant (25) sont :

a) La règle $\Delta S = \Delta Q$,

b) La règle $|\Delta\vec{I}| = \frac{1}{2}$ leptonique pour la partie $\Delta S = 1$,

c) L'absence des transitions avec $\Delta S \geq 2$.

Toutes ces propriétés sont vérifiées expérimentalement, et nous les imposerons à tous les modèles que nous allons construire.

Jusqu'à maintenant, nous n'avons pas introduit de courants neutres, mais expérimentalement nous sommes seulement sûrs de leur absence dans la partie $\Delta S = 1$ du courant (absence de transitions $K_L^0 \not\to \mu^+\mu^-$ et $K^+ \not\to \pi^+\nu\bar{\nu}$). La situation dans la partie $\Delta S = 0$ était inconnue jusqu'aux résultats des dernières expériences de Gargamelle, confirmés à NAL, qui semblent démontrer l'existence des courants neutres sans changement d'étrangeté. Par conséquent, dans la construction de nos modèles, nous imposerons l'absence des courants neutres avec $\Delta S = 1$, mais nous laisserons le choix pour leur introduction à $\Delta S = 0$.

3. INTERACTIONS NON LEPTONIQUES

Ce sont, par exemple, les désintégrations non leptoniques des hypérons, des mésons K, etc.

Nous aurons très peu à dire pour ces interactions. Elles semblent obéir à une régle de sélection $|\Delta \vec{I}| = \frac{1}{2}$ non leptonique dont l'origine est encore un peu mystérieuse. La présence des interactions fortes rend tous les calculs sur ces processus très difficiles. Cependant, la petite valeur de la différence de masse $K_1^0 - K_2^0$ nous montre l'absence des transitions avec $\Delta S \geq 2$.

4. LAGRANGIEN TOTAL -- BOSON INTERMEDIAIRE

En résumant les chapitres précédents, nous constatons que toutes les interactions faibles peuvent être décrites par un Lagrangien phénoménologique de la forme :

$$\mathcal{L}_{eff} = \frac{G}{\sqrt{2}} J_\mu(x) J^{\mu\dagger}(x) , \tag{26}$$

où $J_\mu(x)$ est le courant total faible donné par :

$$J_\mu(x) = \ell_\mu(x) + h_\mu(x) . \tag{27}$$

La simple forme (26) rend compte de tous les phénomènes observés à l'exception de :

a) La règle $|\Delta \vec{I}| = \frac{1}{2}$ non leptonique,

b) La violation de CP aux désintégrations du K_L^0,

c) Les événements de courants neutres récemment observés.

Pourtant, la forme (26) est très arbitraire. D'abord, on voudrait avoir une justification à un niveau plus fondamental du caractère vectoriel des courants; ensuite, comme nous l'avons déjà remarqué, la forme des courants dans (26) n'est pas bien définie à cause de la possibilité d'esfectuer des transformations de Fierz. Pour répondre à ces deux difficultés, les théoriciens ont suggéré, par analogie avec les interactions électromagnétiques, l'existence d'un quantum des interactions faibles, analogue au photon, qui serait une particule vectorielle massive, d'une masse suffisamment élevée pour avoir échappé à la détection jusqu'à ce jour. Les courants $J_\mu(x)$ étant chargés, il nous faut des particules chargées de spin 1. Le Lagrangien (26) s'écrit alors :

$$\mathcal{L}_{eff} = g\, J_\mu(x)\, W^{\mu +}(x) \quad + h.c. \tag{28}$$

où $W^\mu(x)$ est le champ du boson intermédiaire et g est une constante de couplage. Tous les phénomènes décrits par (26) sont obtenus par (28) en second ordre par l'émission et la réabsorption d'un W virtuel. Par exemple, la désintégration du μ est représentée dans les deux cas par les diagrammes de la figure 1 :

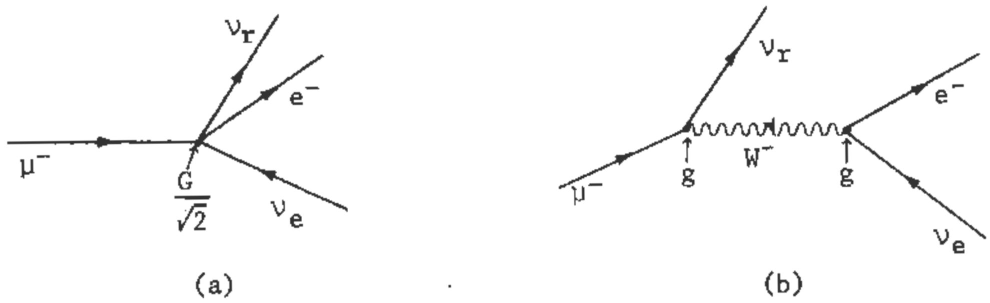

Fig. 1

Si tous les moments de transfert sont petits comparés à la masse du W, l'amplitude de la figure 1(b) se réduit à celle de la figure 1(a) en posant :

$$\frac{G}{\sqrt{2}} = \frac{g^2}{m_W^2} . \tag{29}$$

LECTURE 2 *

I. INTRODUCTION

Weak interaction phenomena are well described by a simple phenomenological model involving a single charged vector boson coupled to an appropriate current.

$$\mathcal{L} = g\ J^{\mu}(x)\ W_{\mu}^{+}(x) + h.c. \tag{1}$$

The weak current $J_{\mu}(x)$ is a sum of two parts, a leptonic part $\ell_{\mu}(x)$ and a hadronic one $h_{\mu}(x)$. They are both of the V - A form and satisfy simple algebraic relations. The leptonic current can be written in terms of the fields of the known leptons as :

$$\ell_{\mu}(x) = \bar{\mu}(x)\ \gamma_{\mu}(1+\gamma_5)\ \nu_{\mu}(x) + \bar{e}(x)\ \gamma_{\mu}(1+\gamma_5)\ \nu_e(x) \tag{2}$$

where we have used the letters μ, e, ν_{μ}, and ν_e to denote the field operators of the corresponding particles. On the other hand no simple expression exists for $h_{\mu}(x)$ in terms of the fields of known hadrons. It may have several forms depending on which particles we consider as elementary. Furthermore, the strong interactions tend to cover up the results of any underlying simplicity. In spite of this uncertainty, if one works on the phenomenological level, one can still obtain a good description of the data by assuming only a general structure and some symmetry properties of the hadronic current. These properties can most easily be exhibited in a simple quark model. Let p(x), n(x), and $\lambda(x)$ represent the fields of the three quarks, then $h_{\mu}(x)$ is given by :

$$h_{\mu}(x) = \cos\theta\ \bar{p}(x)\ \gamma_{\mu}(1+\gamma_5)\ n(x) + \sin\theta\ \bar{p}(x)\ \gamma_{\mu}(1+\gamma_5)\ \lambda(x) \tag{3}$$

where θ is the Cabbibo angle. We emphasize however that the quark fields in (3)

* Ce cours étant déjà rédigé en anglais, nous le publions tel quel pour ne pas retarder la parution.

are not supposed to represent real elementary particles but they serve merely to illustrate the Cabbibo structure and the transformation properties of $h_\mu(x)$.

Taking $J_\mu(x) = \mathcal{L}_\mu(x) + h_\mu(x)$, the Lagrangian of eq. (1) describes completely all known weak interactions and correctly predicts all the observed selection rules with the exception of the $|\Delta\vec{I}| = \frac{1}{2}$ non-leptonic rule. Thus, at the phenomenological level, eq. (1) gives a satisfactory model.

The trouble arises when one wants to take eq. (1) seriously as a field theory and tries to compute the higher order corrections to the lowest order diagrams. Notice that such an attempt is not of pure academic interest, even for present day energies, since there exists at least two known processes, which are forbidden in lowest order, and for which an estimation of the second order effects would have been very instructive. They are the K°_S - K°_L mass difference and the decay $K^\circ_L \to \mu^+ \mu^-$. In a quark language K°_L is a bound state of $(\bar{\lambda} n)$ and $(\bar{n}\lambda)$, therefore the relevant diagrams for these two processes are the ones shown in fig. 1. Unfortunately, even if we neglect all strong interactions associated with the quark lines, these diagrams are not calculable because they are quadratically divergent. One deals with integrals of the form

$$\int \frac{d^4k}{k^2} \to \infty$$

The alert reader might wonder why we worry so much about these divergences since infinities have not stopped theorists from performing wonderful calculations in quantum electrodynamics. The difference is that in Q. E. D. all divergences appear only at certain places, namely the electron and photon self-energies and the radiative corrections to the vertex, and can be absorbed by renormalisation. Of course by doing so we introduce arbitrary constants in the theory and therefore we give up hope of ever being able to predict quantities like the electron mass or the fine structure constant. But once these values are fitted by experiment, all other processes are finite and predictable. Such nice theories are called renormalizable. On the contrary, in our case divergences appear everywhere. If we try to apply the renormalisation program, we would have to introduce arbitrary constants for every process and therefore the theory would lose every predictive power. We call such theories non-renormalizable. To come back to our example, the divergence of the diagrams of fig. 1 means that we cannot predict the second order contribution to the $K^\circ_L \to \mu^+ \mu^-$ branching ratio or the $K^\circ_S - K^\circ_L$ mass difference. One may again ask why the corresponding two-photon exchange diagram in quantum electrodynamics is finite since it looks very much like our troublesome ones of fig. 1. The answer is very simple : the photon propagator can be taken to be $g_{\mu\nu}/k^2$ and therefore behaves, at large momenta, like k^{-2}. The W-propagator however, because of its non-zero mass, is given by

$$\frac{1}{k^2 - m_w^2} \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{m_w^2} \right]$$ which goes asymptotically like a constant. Therefore the same diagram is of the form

$$\int \frac{d^4 k}{k^6} \to \text{finite}$$ in Q. E. D. while infinite in our case.

It is the $k_\mu k_\nu$ part of the W-propagator which has been the source of so much trouble and so many papers in the theory of weak interactions in the last twenty-five years.

II. TOWARDS A BETTER THEORY

Let us now try to solve the following seemingly hopeless problem : is it possible to modify the Lagrangian (1) in such a way that we obtain a renormalizable field theory without upsetting too much its nice agreement with experiment ? And is this possible while still keeping the fundamental vector character of the interaction ? And, last but not least, is it possible to achieve all this in an aesthetically elegant way ? As we shall see, after the works of Veltam, 't Hooft, Lee and Zinn-Justin, the answer to the first two questions is yes ; as for the third, I will let everybody judge according to his personal taste.

In order to gain some insight into the problem, let us first look more closely where the trouble comes in Lagrangian (1). Thus we must study the diagrams of fig. 1. Let us even further simplify the problem by looking at the corresponding purely leptonic processes, for example $\nu+\bar{\nu} \to \nu+\bar{\nu}$. A typical second order diagram is the one in fig. 2 (a) which, as we have already said, is quadratically divergent. An alternative way to say the same thing is to consider calculating this diagram by a dispersion integral over its imaginary part which is proportional to the cross section $\nu + \bar{\nu} \to W^+ + W^-$ shown in fig. 2 (b). A straightforward evaluation of this process shows that it grows asymptotically like

$$\sigma\ (\nu + \bar{\nu} \to W^+ + W^-) \sim \frac{g^2}{m_w^4\, s} \tag{4}$$

thus indicating that the dispersion integral requires two subtractions. A closer examination reveals that this disaster is due to the contribution to the cross section of the longitudinally polarized W's. In fact the cross section for production of transverse bosons behaves like

$$\sigma\ (\nu + \bar{\nu} \to W_T^+ + W_T^-) \sim \frac{g^2}{s} \tag{5}$$

It is when summing over polarizations by using

$$\sum \varepsilon_\mu\, \varepsilon_\nu = g_{\mu\nu} - \frac{k_\mu\, k_\nu}{m_w^2} \tag{6}$$

that we find the trouble-making $k_\mu\, k_\nu$ term responsible for the bad behaviour of eq. (4). Since a four-fermion amplitude must be convergent in a renormalizable theory, we conclude that the behaviour (4) is intolerable and we must find ways to cancel it. Once this is realized, we can easily convince ourselves that the possible ways out are in fact very limited. One has to add new diagrams contributing to the process $\nu+\bar{\nu} \to W^+ + W^-$ in such a way as to cancel the bad asymptotic properties of that of fig. 2 (b). They will necessarily involve new particles since fig. 2 (b) is the only diagram with the existing ones . The new exchanges will be in the s or u channels giving as possibilities the diagrams of fig. 3. Z is a neutral boson and we can easily see that it must have spin equal to one. E has the lepton number of the electron but it has positive charge. It is presumably heavy since it has never been seen up to now. Therefore we conclude that a better theory (if it exists !) must necessarily contain neutral currents and/or heavy, positively charged, leptons. In both cases we can arrange the couplings so that the offending terms coming from fig. 2 (b) are cancelled. This idea can be pursued in other processes of the type $L\bar{L} \to W^+ W^-$ and we are naturally led into a scheme of interrelated couplings characteristic of the Yang-Mills theories. This is not very surprising since there exists an old theoretical prejudice saying that the best behaving theory is the most symmetric one, and the Yang-Mills couplings provide the most symmetric way to couple neutral and charged vector bosons together.

III. MODEL BUILDING

In Professor Itzykson's talk we learnt the systematics of Yang Mills theories as well as the Higgs-Kibble mechanism. We shall try to apply these principles in order to construct realistic renormalizable models of weak interactions.

To begin with, let us remark that we don't know yet whether gauge invariance has anything to do with weak interactions, but we do know that it describes the electromagnetic ones. Therefore it is natural to seek for unified theories of both interactions. We also remind that our only chance was to introduce neutral currents and/or heavy leptons. The first alternative will give us Weinberg's model, the second Glashow and Georgi's, while a combination of both is used in the models of Lee, Prentki, and Zumino.

The essential steps of model building are quite simple and have been given by many authors. We summarize them here :

1) Choose a gauge group G.

2) Choose the fields of the "elementary particles" you want to introduce and their representations. Don't forget to include the Higgs-Kibble scalars Prof. Itzykson told us about.

3) Write the most general renormalizable Lagrangian invariant under G.

4) Choose the "potential" of the Higgs-Kibble scalars so that spontaneous symmetry-breaking occurs, the way it was explained by Prof. Itzykson.

5) Re-write the Lagrangian in terms of the translated fields, choose a suitable gauge and quantize.

We are going to follow this programm for some characteristic models. The reader who is not interested in technical details is advised to skip the constructive part and go directly to the final form, after step 5 is performed. In order to simplify the picture we shall at first rectrict ourselves to the leptonic world.

A) Weinberg's Model

Step 1 : As we said already, this is a model with a neutral leptonic current. Therefore we expect to have at least four vector bosons, namely the photon, the two charged W's and the new neutral one. Since each vector boson corresponds to a generator of G, it follows that the natural group to choose is SU(2) x U(1).

Step 2 : We shall limit ourselves to the known leptons, so let us introduce the electron and its neutrino as a two-component spinor and define its left and right chiral parts :

$$\psi = \begin{pmatrix} \nu \\ e^- \end{pmatrix} \qquad L = \frac{1}{2}(1 + \gamma_5)\ \psi \quad , \quad R = \frac{1}{2}(1 - \gamma_5)\ \psi \tag{7}$$

The muon and its associated neutrino are treated in exactly the same way. Furthermore we need the Higgs-Kibble scalars which are going to break the symmetry and give masses to everybody. We heard in Prof. Itzykson's talk that for each vector boson that acquires a mass, there is a corresponding Higgs-Kibble scalar which become unphysical and decouples. Furthermore there is at end at least one neutral scalar which remains physical, namely the one which developed a non zero vacuum expectation value. Since we must give masses to three vector bosons (the fourth one, the photon, will remain massless), we need at least four scalars which we are going to describe by a 2 x 2 matrix Φ . We now assign transformation properties to all these fields under the group SU(2) x U(1). They are given by :

U(1) :

$$L \to e^{\frac{i}{2} g'\theta(x)} L$$

$$R \to e^{ig'(\frac{1}{2} - \tau_3)\,\theta(x)} R$$

$$\Phi \to \Phi\, e^{ig'\tau_3\theta(x)}$$

SU(2) :

$$L \to e^{ig\vec{\tau}.\vec{\theta}(x)} L$$

$$R \to R$$

$$\Phi \to e^{ig\vec{\tau}\,\theta(x)}\,\Phi \qquad (8)$$

Step 3 : We now write the most general renormalizable Lagrangian invariant under (8) :

$$\begin{aligned} \mathscr{L} = & -\frac{1}{4}\vec{A}_{\mu\nu}.\vec{A}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + \bar{L}\, i\gamma^{\mu}\left[\partial_{\mu} - \frac{i}{2} g' B_{\mu} - ig\vec{\tau}.\vec{A}_{\mu}\right] L \\ & + \bar{R}\, i\gamma^{\mu}\left[\partial_{\mu} - ig'(\frac{1}{2} - \tau_3) B_{\mu}\right] R \\ & - \sqrt{2}\,\bar{L}\,\Phi\, G\, R - \sqrt{2}\,\bar{R}\, G\,\Phi^{+} L. \\ & + \frac{1}{2} \mathrm{Tr} \left| \left[\partial_{\mu}\Phi - ig'\Phi(\frac{1}{2} - \tau_3) B_{\mu} - ig\,\vec{\tau}.\vec{A}_{\mu}\Phi\right] \right|^{2} \\ & - \frac{1}{2}\mu^{2}\,\mathrm{Tr}\,(\Phi\Phi^{+}) - \frac{\lambda}{4}\mathrm{Tr}\,(\Phi\Phi^{+})^{2} \end{aligned} \qquad (9)$$

where

$$\vec{A}_{\mu\nu} = \partial_{\mu}\vec{A}_{\nu} - \partial_{\nu}\vec{A}_{\mu} - g\,\vec{A}_{\mu}\times\vec{A}_{\nu} \qquad (10)$$

$$B_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} \qquad (11)$$

and G is a 2 x 2, real, diagonal, numerical matrix of the form :

$$G = \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix}$$

The Lagrangian (9) is obviously invariant under the gauge transformations (8). A few points should be noticed : (i) the vector bosons $\vec{A}_\mu$ and B_μ are massless as required by gauge invariance. (ii) since L and R have different transformation properties, a fermion mass term of the form $\bar{\psi}\psi = \bar{L}R + \bar{R}L$ is not invariant. Thus the fermions are also massless. (iii) Because of its transformation properties, Φ can be considered as a vector in 0(4) of the form $\Phi = \frac{1}{\sqrt{2}}(\varphi + i\,\vec{\tau}.\vec{\chi})$. It follows that the only invariant is its length of the form $\varphi^2 + \vec{\chi}^2$ and therefore we have only one fourth degree term. (iv) $\mathcal{L}$ contains two coupling constants, g and g'. It has become customary to define an angle $tg\ \theta_w = g'/g$.

In summary, we have succeeded at this step in writing a Lagrangian which has all sorts of nice invariance properties, but it contains massless vector bosons and massless fermions and consequently at first sight, it has nothing to do with physics in general and the weak interactions in particular. Now it is time to pass to :

Step 4 : The classical potential of the Φ-field in (9) is given by :

$$V(\Phi) = \frac{1}{2}\mu^2\, Tr\,(\Phi\,\Phi^+) + \frac{\lambda}{4}\, Tr\,(\Phi\,\Phi^+)^2 \tag{13}$$

The condition for a spontaneous symmetry breaking at the classical level is that V has a minimum for $\Phi \neq 0$. This, for the quadratic form (13), means $\mu^2 < 0$. (The case $\mu^2 = 0$ is marginal. S. Coleman and E. Weinberg have shown that in this case spontaneous symmetry breaking still occurs, only not at the classical level but when radiative corrections are taken into account). The minimum occurs at a value

$$<\Phi> = \frac{v}{\sqrt{2}} \quad , \quad v^2 = -\frac{\mu^2}{\lambda} \tag{14}$$

Step 5 : We now define a new field Φ , with zero vacuum expectation value, by letting

$$\Phi \to \Phi + \frac{v}{\sqrt{2}} \tag{15}$$

where v is proportional to the unit matrix and its value is given by (14). The result of this translation on the Lagrangian (9) is rather lengthy but we shall single out the most interesting terms :

(i) The Yukawa couplings on the fourth line of eq. (9) give, under the translation (15) :

$$- v \bar{L} G R - v \bar{R} G L = - v G_1 \bar{\nu} \nu - v G_2 \bar{e} e \tag{16}$$

where the form (12) of G has been used. We therefore see that we have generated fermion mass terms. Since we want to keep the neutrino massless, we choose :

$$G_1 = 0 \Rightarrow m_\nu = 0 \quad , \quad m_e = v G_2 \tag{17}$$

(ii) The vector boson-Φ couplings on the fifth line of (9) give :

$$\frac{1}{8} g^2 v^2 \vec{A}_\mu^{\,2} + \frac{1}{8} g'^2 v^2 B_\mu^{\,2} + \frac{1}{4} g g' B^\mu . A_\mu^3 \tag{18}$$

The expression (18) shows that we have also generated mass terms for the vector bosons, however there exists a non-diagonal mass matrix between the two neutral ones, B_μ and A_μ^3, given by :

$$M_{AB} = \frac{1}{8} v^2 \begin{pmatrix} g'^2 & g g' \\ g g' & g^2 \end{pmatrix} \tag{19}$$

It has obviously one eigenvalue equal to zero and the corresponding boson will be identified with the photon. By diagonalizing (19) we write (18) as :

$$\frac{1}{4} g^2 v^2 W_\mu^+ W^{\mu-} + \frac{1}{8} v^2 (g^2 + g'^2) \quad Z_\mu^2 \tag{20}$$

where we have put :

$$W_\mu^{\pm} = A_\mu^{\pm} \quad , \qquad \Longrightarrow \quad m_{W^\pm} = \frac{1}{2} g v$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g' B_\mu + g A_\mu^3) \Longrightarrow m_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (- g B_\mu + g' A_\mu^3) \Longrightarrow m_A = 0 \qquad (21)$$

The relations (20) and (21) show that A_μ is the photon field with zero mass, $W_\mu^{\pm}$ are the conventional, charged intermediate bosons with mass $\frac{1}{2}$ g v , and Z_μ is a neutral vector boson with mass $\frac{1}{2}$ v $\sqrt{g^2 + g'^2}$.

(iii) The masses of the scalar mesons are also changed. The last line of eq. (9) gives :

$$\Phi = \frac{1}{\sqrt{2}} (\varphi + i \vec{\tau}.\vec{\chi}) \Longrightarrow m_\varphi^2 = 2 \lambda v^2 \quad , \quad m_\chi = 0 \qquad (22)$$

i. e. the three χ -fields are massless. In Prof. Itzykson's talk we saw that they correspond to the would-be Goldstone bosons and they are actually decoupled from the rest of the Lagrangian. Their degrees of freedom were used to create the extra polarization states required for the transition from massless to massive vector bosons.

(iv) The couplings of the photon are the usual electromagnetic ones. The electron charge is given by :

$$\frac{g g'}{\sqrt{g^2 + g'^2}} \bar{e} \gamma_\mu e A_\mu \Longrightarrow e = \frac{g g'}{\sqrt{g^2 + g'^2}} = g' \cos \theta_W \qquad (23)$$

The photon-charged vector boson couplings correspond to a gyromagnetic ratio equal to two. One can show (see e. g. Prof. Lautrup's report in R. de Moriond 1973) that this value gives the best behaviour of the theory.

(v) The ordinary weak interactions are of the form :

$$\frac{g}{2\sqrt{2}} \bar{\nu} \gamma^\mu (1 + \gamma_5) e W_\mu^+ + \text{h.c.} \Rightarrow \frac{G}{\sqrt{2}} = \frac{g^2}{8 m_{W^\pm}^2} \qquad (24)$$

where $\frac{G}{\sqrt{2}}$ is the Fermi coupling constant. Combining (23), (24) and (21), we find :

$$m_{W^{\pm}} \geq 37.5 \text{ GeV} \qquad\qquad m_Z \geq 75 \text{ GeV} \qquad\qquad (25)$$

which is one of the most striking predictions of these theories.

(vi) Finally we notice the couplings of the neutral intermédiate vector boson Z_μ which provides the characteristic feature of this model :

$$\frac{\sqrt{g^2+g'^2}}{4}\left\{\bar{\nu}\gamma_\mu(1+\gamma_5)\nu-\bar{e}\gamma_\mu\left[\frac{g^2-3g'^2}{g^2+g'^2}+\gamma_5\right]e\right\}Z^\mu \qquad (26)$$

The experimental implications of (26) are obvious. There is a clear prediction of this model, namely the existence of neutral leptonic currents. They are either of the form $\bar{\nu}\nu$ or $\bar{e}e$ ($\bar{\mu}\mu$). The first ones are more likely to be observed in high energy experiments and we have all heard the positive results of Gargamelle and N. A. L. (for a review see the report of P. Musset at the 2nd Aix-en-Provence Conference 1973). However, even the $\bar{e}e$ neutral leptonic currents can give measurable effects in atomic physics experiments.

There are all sorts of other couplings induced by the translation but we won't give them explicitly. They can be trivially computed from the Lagrangian (9). Notice only that there remain Yukawa couplings of the φ -meson to the leptons with a strength proportional to the fermion mass. Notice also that the φ -mass is not restricted by the theory.

In conclusion we have presented a unified theory of weak and electromagnetic interactions of leptons consistent with all present day experiments and which can be extended to a renormalizable theory (the extension requires the introduction of hadrons). This has been achieved with the introduction of two extra neutral intermediaries, a vector and a scalar. Their couplings are predicted and they provide clear experimental tests of the model.

B) The Model of Georgi and Glashow

Georgi and Glashow asked themselves the following question : Suppose that experiments show that there are no neutral lepton currents, thus proving unambiguously that Weinberg is wrong. Should one abandon the whole idea or can one still be in business ? In other words, can one build a model having all the nice properties of renormalizability in which the only neutral current is the electromagnetic one ? It turns out that the answer is yes, and we saw already that in this case one expects to be forced to introduce heavy leptons.

The strategy for the construction of the model is the same as previously, so we won't go through in so much detail. We choose SO(3) as the gauge group thus having a triplet of vector bosons. The neutral member is the photon, the other two the usual intermediaries of the weak interactions. It turns out that the most economical solution requires the introduction of four heavy leptons, two electronic and two muonic ones. The right- and the left-handed leptons form the triplets of SO(3) :

$$\vec{\psi}_{e_R} = \frac{1}{2}(1-\gamma_5)\begin{bmatrix} X^+ \\ X^\circ \\ e^- \end{bmatrix} , \quad \vec{\psi}_{e_L} = \frac{1}{2}(1+\gamma_5)\begin{bmatrix} X^+ \\ X^\circ \cos\beta + \nu \sin\beta \\ e^- \end{bmatrix} \tag{27}$$

where X^+ and X° are heavy leptons having the leptonic number of an electron. The angle β gives the mixing of the two neutral states the X° and the neutrino. Furthermore we introduce the SO(3) singlets.

$$S_{e_L} = \frac{1}{2}(1+\gamma_5)\,(X^\circ \sin\beta - \nu\cos\beta) \quad , \quad S_{e_R} = \frac{1}{2}(1-\gamma_5)\,\nu \tag{28}$$

and define

$$\vec{\psi} = \vec{\psi}_{e_L} + \vec{\psi}_{e_R} \tag{29}$$

Again, the muons are treated in exactly the same way with the introduction of two new states, Y^{+} and Y°, and the definition of the triplet $\vec{\psi}_{\mu}$ with the analogue of eq. (29). Notice that if β' is the corresponding mixing angle between Y° and ν_{μ}, one expects a priori to have $\beta \neq \beta'$. Finally we include a triplet of Higgs-Kibble scalars $\vec{\Phi}$. The Lagrangian can now be written as :

$$\begin{aligned}
\mathcal{L} = & \, i \, \bar{\vec{\psi}}_e \not{\partial} . \vec{\psi}_e - m_0 \bar{\vec{\psi}}_e . \vec{\psi}_e + i \, \bar{s}_{e_L} \not{\partial} s_{e_L} + i \, \bar{s}_{e_R} \not{\partial} s_{e_R} \\
& - ie \, \vec{W}_{\mu} . (\bar{\vec{\psi}}_e \, x \gamma_{\mu} \vec{\psi}_e) - \frac{1}{4} \vec{W}_{\mu\nu} . \vec{W}^{\mu\nu} \\
& + \vec{\Phi} . \left[i g_1 \bar{\vec{\psi}}_e x \vec{\psi}_e + (g_3 \bar{\vec{\psi}}_e s_{e_L} + \text{h.c.}) \right] \\
& + \frac{1}{2} (\partial_{\mu} \vec{\Phi} + e \vec{W}_{\mu} \times \vec{\Phi})^2 \\
& - \frac{1}{2} \mu^2 \vec{\Phi} . \vec{\Phi} - \frac{\lambda}{4} (\vec{\Phi} . \vec{\Phi})^2
\end{aligned} \qquad (30)$$

Several points should be noticed here : (i) the right-handed neutrino is not coupled, so the last term of the first line is superfluous.

(ii) Even before symmetry breaking the fermions are not massless since the term $\bar{\vec{\psi}}_e . \vec{\psi}_e$ is an invariant. The W's, on the other hand,still have zero mass.

(iii) There is only one coupling constant between the W's and the other particles. This means that all particles in this theory must have integral electric charges, therefore this model does not allow for any fractionally charged quarks. This peculiarity is due to the fact that the group SO(3) has no abelian sub-group.

(iv) The coupling of the W's to the fermions does not contain any explicit axial currents. Such couplings are called "vector-like" and they have the nice property of not suffering from anomalies in the Ward identities.

The spontaneous symmetry breaking is performed in exactly the same way as before by choosing $\mu^2 < 0$. After translation, we find :

(i) The charged W's acquire a mass $m_W = ev$ where $v = \sqrt{-\mu^2/\lambda}$.

The neutral vector boson remains massless and can be identified to the photon.

(ii) All fermions but the neutrino get different masses. One finds a relation among them of the form :

$$2 \, m \, (X^{\circ}) \cos \beta = m \, (X^{+}) + m \, (e^{-}) \qquad (31)$$

(iii) Again, the charged Φ's become massless and can be shown to decouple.

(iv) The photon is coupled to fermions as usual :

$$e A_\mu \left[\bar{X}^+ \gamma^\mu X^+ - \bar{e} \gamma^\mu e \right] \tag{32}$$

(v) The W-fermion couplings are given by :

$$\begin{aligned} - \frac{e}{2} W_\mu^+ \Big\{ & \bar{X}^+ \gamma^\mu (1 - \gamma_5) X^\circ - \bar{X}^\circ \gamma^\mu (1 - \gamma_5) e \\ & + \bar{X}^+ \gamma^\mu (1 + \gamma_5) X^\circ \cos \beta - \bar{X}^\circ \gamma^\mu (1 + \gamma_5) e \cos \beta \\ & + \bar{X}^+ \gamma^\mu (1 + \gamma_5) \nu \sin \beta - \bar{\nu} \gamma^\mu (1 + \gamma_5) e \sin \beta \Big\} \\ & + \text{h.c.} \end{aligned} \tag{33}$$

The usual leptonic weak interactions are due to the last term. All the others involve the heavy leptons. The Fermi coupling constant is given by :

$$\frac{G}{\sqrt{2}} = \frac{1}{4} \frac{e^2 \sin^2 \beta}{m_W^2} \Longrightarrow m_W = 53 \text{ GeV} \sin \beta \tag{34}$$

Therefore in this model $m_W \leq 53$ GeV.

A final remark is in order : as we already mentioned, this theory has the attractive feature of imposing automatically the universality of the electromagnetic interactions since the different particles cannot have arbitrary charges. The price is that we lose the weak universality because the analogue of eq. (34) for the muons will involve the angle β' which, as we said previously, is not restricted to be equal to β by the theory. Therefore the weak universality must be enforced by hand by imposing $\beta = \beta'$.

C. The Models of Lee, Prentki and Zumino

These authors remarked that one does not have to kill all the neutral currents of Weinberg's model in order to be safe. It is sufficient to kill the $\bar{\nu}\nu$

part since it is the only one which is likely to be tested experimentally in the near future. It turns out that this is again possible and in more than one way. The resulting models have both neutral currents (but not in the $\bar{\nu}\,\nu$ form) and heavy leptons.

As an example, let us introduce only two heavy positively charged leptons, one associated to the electron and the other to the muon. We are going to use Weinberg's system of vector bosons, namely a triplet $\vec{A}_\mu$ and a singlet B_μ. We then define a triplet of left handed fermions.

$$\vec{\psi}_{e_L} = \frac{1}{2}(1+\gamma_5)\begin{bmatrix} X^+ \\ \nu \\ e^- \end{bmatrix} \tag{35}$$

with all right singlets of SU(2). The coupling of $\vec{\psi}_{e_L}$ to $\vec{A}_\mu$ is like in Georgi and Glashow :

$$g\,\vec{A}_\mu \cdot (\bar{\vec{\psi}}_{e_L}\, X\gamma^\mu\, \vec{\psi}_{e_L}) \tag{36}$$

therefore A_μ^3 is coupled only to charged fermions. Let us couple similarly B_μ but only to right-handed leptons :

$$g' B_\mu \left[\bar{X}^+ \gamma^\mu (1-\gamma_5) X^+ - \bar{e}\,\gamma^\mu (1-\gamma_5)\, e\right] \tag{37}$$

Since none of B_μ and A_μ^3 is coupled to neutrinos, it follows that their linear combinations A_μ and Z_μ won't do so either. Again the photon (A_μ) will be coupled only to the e. m. current and the neutral vector boson Z_μ will have coupling only to $\bar{e}e$ and $\bar{X}^+ X^+$.

We must also introduce Higgs-Kibble scalars and it turns out that in this point the model is not so economical. In order to make the extension to hadrons possible, one needs nine independent scalar fields and in the end one is left with six physical scalar particles. Like in Weinberg's model, the weak universality is automatic once the electromagnetic one is imposed by hand.

D) Extension to Hadrons

We finally discuss briefly the problems connected with the extension of these ideas to the hadronic world. Since we want to use the field theory language, we face back the old question of deciding which hadrons are elementary. We therefore anticipate a certain degree of arbitrariness. Again all arguments will be presented in a quark model framework, thus putting the emphasis on the symmetry principles of the currents rather than on detailed dynamical assumptions.

A first observation is going to be very crucial : the traditional SU(3) scheme of strong interactions is incompatible with the kind of models we have discussed so far. This can be seen quite easily by re-writing the weak hadronic current $h_\mu(x)$ given by eq. (3). It is actually convenient to use a compact notation by defining a quark spinor q and a 3 x 3 matrix C,

$$q(x) = \begin{pmatrix} P(x) \\ n(x) \\ \lambda(x) \end{pmatrix} \qquad C = \begin{pmatrix} 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{38}$$

where θ is the Cabbibo angle. Then eq. (3) can be written as :

$$h_\mu(x) = \bar{q}(x)\,\gamma_\mu\,(1+\gamma_5)\,C\,q(x) \tag{39}$$

Let us call Q_W the charge associated with $h_\mu(x)$ and Q_W^+ the one associated with $h_\mu^+(x)$. By commutation we find the charge Q°_W which will correspond to the neutral current

$$h^\circ_\mu(x) = \bar{q}(x)\,\gamma_\mu\,(1+\gamma_5)\,C^\circ\,q(x) \tag{40}$$

where

$$C^{\circ} = \left[C, C^{+} \right]$$

Using (38) we compute C° and find :

$$C^{\circ} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\cos^2\theta & -\sin^2\theta \\ 0 & -\frac{1}{2}\sin^2\theta & -\sin^2\theta \end{pmatrix} \qquad (41)$$

The expressions (40) and (41) form the basis of our argument. They show that the current h_{μ}° is not the e. m. current, therefore the SU(3) quark model cannot supplement the Georgi and Glashow leptonic model. This result was known already since, as we said before, Georgi and Glashow cannot accomodate fractionally charged quarks. However eq. (41) tells us that the other models are no good either, since C° contains off-diagonal terms therefore coupling $\bar{\lambda}n$ to Z_{μ}. In other words a model based on SU(3), as the symmetry of the strong interactions, predicts the existence, at lowest order, of neutral leptonic currents coupled to strangeness changing decays and of $\Delta S = 2$ non leptonic transitions. They are both definitely ruled out by experiment from $K^{\circ}_{L} \rightarrow \mu^{+} \mu^{-}$ branching ratio and the $K^{\circ}_{L} - K^{\circ}_{S}$ mass difference. We conclude that the only way out is to enlarge the symmetries of the strong interactions into a group larger than SU(3). Of course there are several ways of doing so, each one introducing new symmetries into the hadron model, but they all share the prediction of new hadronic, as yet unobserved states. We shall call such states "charm" and again, their existence provides a general test of these ideas.

As an example of a simple solution we present the construction of Glashow, Illiopoulos and Maiani. They enlarge the strong interaction symmetries from SU(3) to SU(4) by adding a fourth quark p' which has the same electric charge as p but carries an extra quantum number, the "charm". In this model the weak hadronic current is given by the analogue of eq. (39) but with q and C written as :

$$q(x) = \begin{pmatrix} P'(x) \\ P(x) \\ n(x) \\ \lambda(x) \end{pmatrix} \qquad C = \begin{pmatrix} 0 & \begin{matrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{matrix} \\ 0 & 0 \end{pmatrix}$$

(42)

The advantage of eq. (42) is that now the neutral current contains $C^{\circ} = \left[C, C^{\dagger} \right]$ which equals :

$$C_0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \qquad (43)$$

i. e. the neutral currents are coupled only to strangeness-conserving transitions where the existing data are encouraging.

The SU(4) mechanism of cancelling the unwanted transitions can be applied to other quark models as well. An example is the Han-Nambu three triplet model which now must be extended to a three quartet one by adding a p' to each column. It has been shown by Bouchiat, Iliopoulos and Meyer that both the Han-Nambu SU(4) model and the fractionally charged model we discussed previously, can be incorporated into Weinberg's leptonic model without any conflict with experimental data. Notice however that in the latter case of fractionally charged quarks we must again use three quartets, either by introducing parastatistics or by using Gell-Mann's three coloured quarks. This is imposed upon us by requirement of having normal Ward identities for the axial currents, which in turn is a necessary condition for renormalizability. I find this rather appealing because we arrive to the same result starting from completely different requirements. I remind you that the parastatistics or coloured quark models were introduced in the framework of SU(3), independently of weak interactions, in order to allow for the quarks to satisfy the spin-statistics theorem and to predict the correct value for the $\pi^0 \rightarrow 2\gamma$ decay rate.

It turns out that Weinberg's model is the only one which accepts fractionally charged quarks. We saw it already for the Georgi and Glashow one, and one can show that the Lee, Prentki and Zumino models are not renormalizable but with integrally charged quarks because otherwise they have anomalous Ward identities.

The Georgi and Glashow model requires at least eight basic quarks with a symmetry group of the strong interactions SU(8). However one can express the model also in terms of the Han-Nambu nine quarks in which case the strong symmetry group is SU(3) x SU(3). However, in the latter case several complicated conditions must be imposed by hand thus sacrifying a great deal of elegance and reducing the credibility of an otherwise very beautiful model.

Let us summarize the whole story. At the beginning of the second chapter we asked ourselves three fundamental questions. We showed that the answer to the first two is a triumphant yes. There exist several models which, without contradicting the present experimental data, combine the fundamental vector character of the interaction with renormalizability. This is by itself a very important achievement, and, in my opinion, it has come to stay. Either in one of its present forms, or embedded in a larger and more consistent scheme, the ideas exposed here will be part of our life for quite a few years ahead, and will direct our research, both experimental and theoretical, towards a better understanding of the nature of weak interactions. On the other hand, the answer to our third question is less obvious. The fact that we have several models means that none of them imposes itself by elegance or aesthetic beauty. Past experience in physics has taught us that this situation implies that probably none of them is correct and the good model still remains to be discovered.

REFERENCES

For a complete list of references, see B. W. Lee, rapporteur's talk at the Batavia Conference and S. Weinberg's talk at the Aix-en-Provence Conference 1973.

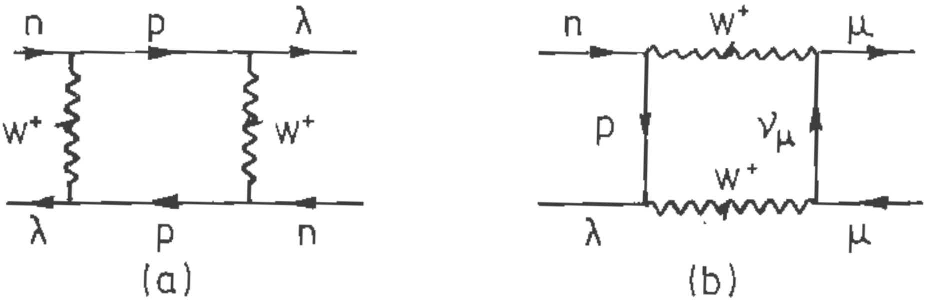

Fig. 1

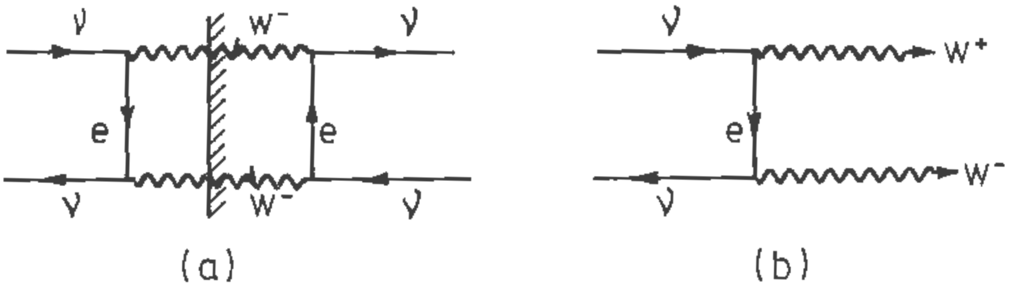

Fig. 2

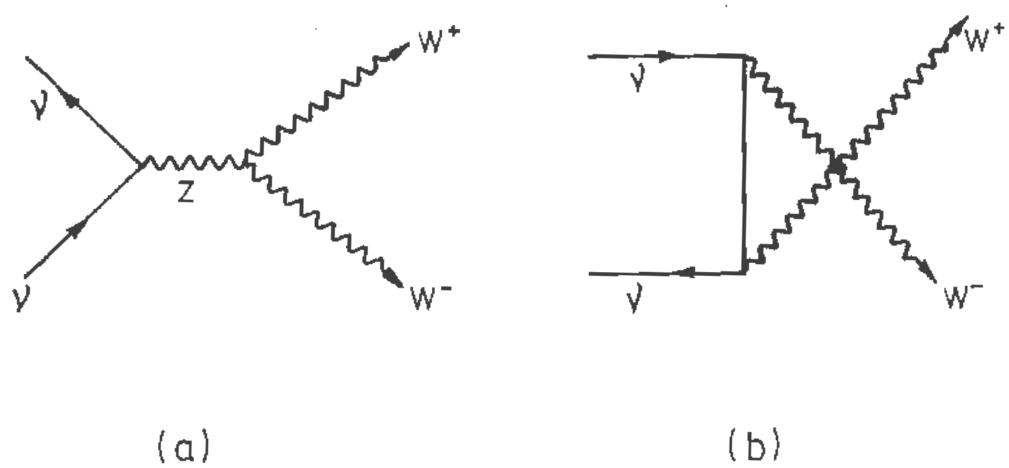

Fig. 3